Image Compression based on Empirical Mode Decomposition

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Abstract

Previous work on Empirical Mode Decomposition in two dimensions typically generates a residue with many extrema points. In this paper we propose an improved method to decompose an image into a number of Intrinsic Mode Functions and a residue image with a minimum number of extrema points. A method of using overlapping 7x7 blocks is introduced to overcome blocking artifacts and variable sampling of the blocks is used to reduce the number of parameters to represent the image. We introduce the concept of empiquency, short for empirical mode frequency, to describe the signal oscillations, since traditional frequency concept is not applicable in this work. All this together is used for Image Compression. This is done blockwise using the non-uniformly located extrema points of the IMF to steer the uniform sampling rate of the block and a modified SPIHT coder is used on high-empiquency blocks.

1 Introduction

Many methods exist to simultaneously analyse signals in the time- and frequency-domain, Short-Time Fourier Tranform , Wigner distribution [2] and Wavelets [10] among others. All these methods are based on the expansion of the signal into a set of basis functions where the basis functions are defined by the method. The concept of the Empirical Mode Decomposition (EMD) [4] is to expand the signal into a set of functions defined by the signal itself, the Intrinsic Mode Functions (IMF). The signal is decomposed into a redundant set of signals denoted IMF and a residue. Adding all the IMFs together with the residue reconstructs the original signal without information loss or distortion. In the two-dimensional case, where the adaptive spatial frequency separation presents a challenge, the EMD enables many new approaches for image processing applications. Some efforts to implement the EMD in two dimensions has been published [8][9] but the methods typically generates a residue with many extrema points.

In this paper we present an improved method for the use in image processing applications [6] that can decompose the image into a number of IMFs and a residue with none, or with only a few extrema points. We also present an image coder using the blockbased variable sampling [7] of the IMF and a modified SPIHT coder together.

2 EMD

The EMD is a highly adaptive decomposition. It provides a decomposition method that analyses the signal locally and separates the component holding the locally highest frequencies from the rest into a separate IMF. Within this IMF both high and low empiquencies can coexist at different times. The key word is *locally*.

Time-frequency analysis by means of HHT, or EMD separately, has been done in several areas such as analysis of ocean waves, seismology, and one-dimensional analysis of SAR images [4][5][12]. The behaviour of the EMD as a filter bank is highlighted by [3] in the analysis of noise.

An IMF is characterized by some specific properties. One is that the number of zero crossings and the number of extrema points is equal or differs only by one. Another property of the IMF is that the envelopes defined by the local maxima and minima, respectively, is locally symmetric around the envelope mean.

To find the first IMF, start with the image itself as input signal $in_{11}(m, n) = x(m, n)$. The first index is the IMF number, l=1..L, and the second index is the iteration number, k=1..K, in the sifting process. m and n represents the two spatial dimension. To find the next IMF, use the residue corresponding to the previously found IMF as input signal $in_{21}(m, n) = r_1(m, n)$.

The sifting process to find the IMFs of a signal *x(m,n)*, comprises the following steps:

(1) Find the positions and amplitudes of all local maxima, and find the positions and amplitudes of all local minima in the input signal $in_{lk}(m, n)$.

(2) Create the upper envelope by spline interpolation of the local maxima and the lower envelope by spline interpolation of the local minima, denoted $e_{max}(m, n)$ and $e_{min}(m, n)$.

(3) For each position *(m,n)*, calculate the mean of the upper envelope and the lower envelope.

$$em_{lk}(m, n) = \frac{e_{max}(m, n) + e_{min}(m, n)}{2} \qquad (1)$$

The signal $em_{lk}(m, n)$ is referred to as the envelope mean.

(4) Subtract the envelope mean signal from the input signal

$$h_{lk}(m, n) = in_{lk}(m, n) - em_{lk}(m, n) \qquad (2)$$

This is one iteration of the sifting process. The next step is to check if the signal $h_{lk}(m, n)$ from step 4 is an IMF or not. The process stops when the envelope mean signal is close enough to zero.

$$|em_{lk}(m, n)| < \varepsilon \qquad \forall(m, n) \qquad (3)$$

The reason for this choice is that forcing the envelope mean to zero will guarantee the symmetry of the envelope and the correct relation between the number of zero crossings and number of extremes that define the IMF, without actually counting the zero crossings and extrema points, or check for symmetry.

(5) Check if the mean signal is close enough to zero, based upon the stop criterion. If not, repeat the process from step 1 with the resulting signal from step 4 as the input signal a sufficient number of times.

$$in_{l(k+1)}(m, n) = h_{lk}(m, n) \qquad (4)$$

When the stop criterion is met, *k=K*, the IMF is defined as the last result from (4).

$$c_l(m, n) = h_{lK}(m, n) \qquad (5)$$

After the IMF $c_l(m, n)$ is found, define the residue $r_l(m, n)$ as the result from subtracting this IMF from the input signal.

$$r_l(m, n) = in_{l1}(m, n) - c_l(m, n) \qquad (6)$$

(6) The next IMF is found by starting over from step 1, now with the residue as the input signal.

$$in_{(l+1)1}(m, n) = r_l(m, n) \qquad (7)$$

The steps (1) to (6) can be repeated for all the subsequent r_l. The EMD is completed when the residue, ideally, does not contain any extrema points. This means that it is either a constant or a monotonic function. The signal can be expressed as the sum of IMF:s and the last residue

$$x(m, n) = r_L(m, n) + \sum_{j=1}^{L} c_j(m, n) \qquad (8)$$

The EMD in two dimensions relies on proper spline interpolation in two dimensions. The problem is to fit a surface to the two-dimensional scattered data points representing the extrema points. A suggestion for two-dimensional EMD is to use thin-plate smoothing spline interpolation [1]. The determination of the smoothing spline involves the solution of a linear system with as many unknowns as there are extrema points. One of the main objections for using spline interpolation in the EMD for two-dimensional signals such as images is that the borders cause too much problems. The set of extrema points is very sparse and since the interpolation methods only interpolate between points, the borders need special care. The trick is to add extra data points at the borders to the set of extrema points. The extra points are placed at the corners of the image and some additional points at the border equally spaced between the corners. Without these extra points, the areas not covered by the interpolation traverses into the image in the sifting process. This method turns out to successfully decompose an image into its IMFs and a smooth residue with none or only a few extrema points.

It should be noted that all the IMFs are locally zero mean while the DC level of the signal is contained in the residue. All the IMFs have the very special property of having only one extrema between two zero-crossings in any direction. This will be used in the next section to get a more efficient representation.

3 Coding using Variable Sampling of Overlapping Blocks

In this part of the paper we present a new method to code an image. It uses the empiquency of the first IMF to steer the choise of coding method used in the different blocks of the image. First the important components are presented.

3.1 Empiquency

The EMD is a truly empirical method, not based on the Fourier frequency approach but related to the locations of extrema points and zero-crossings. Based on this we use the concept of *empiquency*[7], short for empirical mode frequency, instead of a traditional frequency measure to describe the signal oscillations. The measure of empiquency is defined as “One half the reciprocal distance between two consecutive extrema points”. Areas with many extrema points close together have high empiquency while areas with a sparse set of extrema points have low empiquency.

3.2 Noise Reduction

For signals with moderate signal-to-noise ratio, we can assume that the noise is of lower amplitude than the extrema points. Yet, the noise will generate extrema points. We can reduce the noise by setting the extrema points with low amplitude to zero. Most of the pixels are already zero because they are not extrema points. If we let the coefficients with absolute amplitude lower than a suitable threshold be set to zero, the remaining coefficinets represent only signifficant extrema points with respect to the zero zone value. In figure 1 the zero zone is 10.

3.3 Variable Sampling of overlapping blocks

In the implementation we choose to use overlapping blocks of size 7x7 pixels. This is to minimize the artifacts from the blocking and to further reduce the samples used to represent the IMF. The corners of the block is allways represented. The overlapping pixels in two concatenated blocks is the same, but used twice, this will ensure that the concatenated blocks have the same values at the edge pixels. The overlapping pixels will only belong to one of the blocks when patching and counting for bitrate.

This paper concerns the issue of reducing the number of parameters required to represent the image. This is done through variable sampling of overlapping blocks [7] and we use the empiquency to steer the sampling rate for each block individually. Reconstruction of the block is done with spline interpolation of the samples.

3.4 Modified SPIHT Coder

The SPIHT coder used here is a matlab implementation. Modifications to the standard SPIHT scheme [11] is done by means of filter choise and entropy coding of the bitstream. The Haar filter is used and no arithmetric coding of the bit stream is done. The bitstream is left uncompressed at this stage to make further modifications possible. The bitrate measure is the actual size of the bitstream.

3.5 The VS coder

In the first IMF there are areas where every pixel is an extrema point, thus the maximum empiquency is 0.5, meaning that the image cannot be reconstructed without distortion if it is subsampled. Due to the different empiquencies in the different parts of the IMF, the subsampling can be different in different parts of the image. We use the empiquency of the first IMF to decide which method is used on the block. The high empiquency blocks which cannot be subsampled is coded with a modified SPIHT coder. We force the modified SPIHT coder to perform better than 30 dB on each block and choose the lowest bitrate of the candidate codings with performance better than this level to be used in the VS coder.

Each block is represented by its sampled coefficients or its SPIHT bitstream. To each blockrepresentation a two-bit header is added. This header indicates which method is used to code the block and at the same time it indicates where the next block starts in the bitstream. The coefficients from the blocks coded with the variable sampling is quantized and fixlength coded according to the quantisation level. Different size on the zero zone in the noise reduction step allows for different amount of interpolation blocks in relation to SPIHT blocks.

Figure 1: a) The extrema points with the smallest coefficients set to zero

4 Result

We show that an image can be decomposed down to a residue with a minimum of extrema points. The Lenna image is shown in figure 2a, and is decomposed with the EMD method propsosed in this paper. The image's four IMFs and their corresponding residues are shown in figure 2b-2i. The last residue has only a very few extrema points.

INn odter to evaluate the performance of themodified SPIHT coder, the Lenna image is coded with this coder without the blocking procedure. The result is 33.3 dB with 1.8 bpp and 24.4 dB at 0.66 bpp. The modified SPIHT is represented with the star curve in figure 4, and the reconstructed images is shown in figure 3a and 3b. When we code the image with the VS coder, using the variable sampling together with the modified SPIHT, we get the result shown in figure 3. Vscoding zero zone 10 generates 31.2 dB at 0.57 bpp and 24 dB at 0,42 bpp. Vscoding zero zone 1 results in 35.5 dB at 0.57 bpp and Vscoding zero zone 20 gives a result of 29.0 dB at 0.57 bpp. These results are shown in the rest of the curves in figure 4 and the reconstructed images are presentad in figure 3c-3f.

5 Conclusion

Previous work on Empirical Mode Decomposition in two dimensions typically generates a residue with many extrema points. In this paper we propose an improved method to decompose an image into a number of Intrinsic Mode Functions and a residue image with a minimum number of extrema points. We have shown how to decompose a full image EMD into a number of

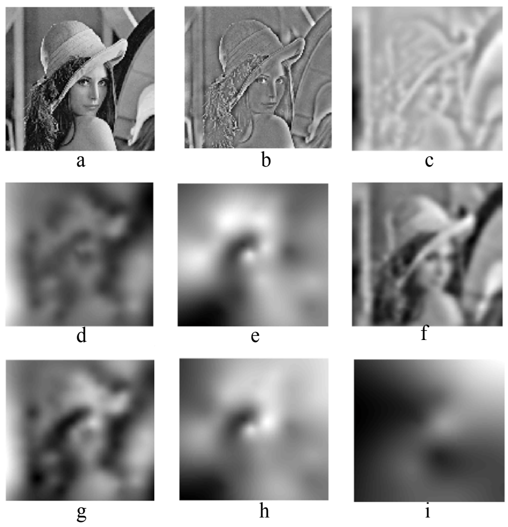

Figure 2: a) original image, b) first IMF, c) second IMF, d) third IMF, e) fourth IMF, f) first residue, g) second residue, h) third residue, i) fourth residue.

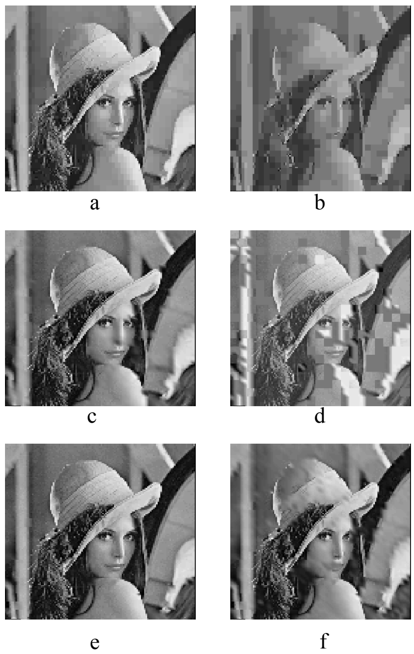

Figure 3: a):Spiht coded to 33.3 dB 1.8bpp. b):Spiht coded to 24.4 dB 0.66 bpp. c)Vscoding zero zone 10, 31.2 dB, 0.57 bpp. d):Vscoding zero zone 10, 24dB 0,42 bpp. e):Vscoding zero zone 1, 35.5 dB, 0.57 bpp. f):Vscoding zero zone 20, 29.0 dB, 0.57 bpp

IMFs and a residue image with no, or a minimum number of, extrema points. The value of ε in the stop criterion affects the EMD in such a way that if it is not small enough there will not be a sufficient number of IMFs to sepaprate all intrinsic modes in the signal. On the other side, if the number ε is too small the iterations vill take long time. The concept of empiquency, short for empirical mode frequency, is introduced to describe the signal oscillations, since traditional frequency concept is not applicable in this work. We have proposed a method for variable sampling of the two-dimensional EMD. A method of using overlapping 7x7 blocks is introduced to overcome blocking artifacts and to further reduce the number of parameters to represent the image. All this together is used for Image Compression. This is done blockwise using the non-uniformly located extrema points of the IMF to steer the uniform sampling rate of the block. Improvements to the scheme would be to further modify the SPIHT coder, regarding header length and use better entropy coding on the bitstream.

6 References

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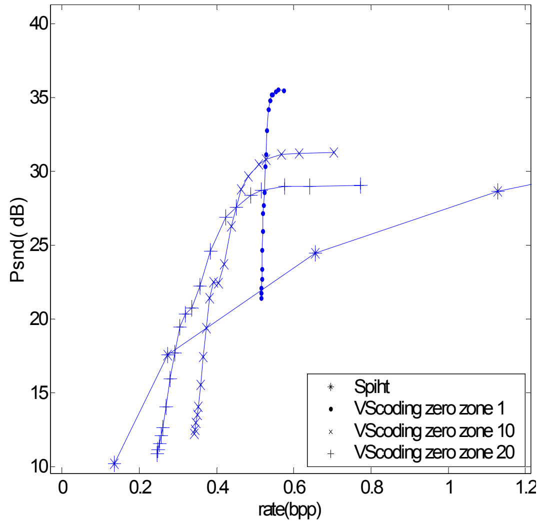

Figure 4: Result from the VS coder and SPIHT coder applied on the Lenna128x128

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